Table of Laplace Transforms

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt, \quad s > 0$$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}} \text{ if } n \geq 0 \text{ is an integer}$$

$$\mathcal{L}\left\{t^p\right\} = \frac{\Gamma(p+1)}{s^{p+1}},\ p > -1$$

$$\mathcal{L}\{\sin(at)\} = \frac{a}{s^2+a^2}$$

$$\mathcal{L}\{\cos(at)\} = \frac{s}{s^2+a^2}$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}, \quad s > a$$

$$\mathcal{L}\{f(\alpha t)\} = \frac{1}{\alpha}F\left(\frac{s}{\alpha}\right), \quad \alpha > 0$$

$$\mathcal{L}\{e^{at}f(t)\} = F(s-a), \quad s > a$$

$$\mathcal{L}\{u(t-a)f(t-a)\} = e^{-as}F(s), \quad s > a \geq 0$$

$$\mathcal{L}\left\{f^{(n)}(t)\right\} = s^n\mathcal{L}\{f(t)\} - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0), \quad n \geq 0$$

$$\mathcal{L}\{t^n f(t)\} = (-1)^n F^{(n)}(s) \equiv (-1)^n \frac{d^n}{ds^n}F(s), \quad n \geq 0$$

$$\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(x)\,dx$$

$$\mathcal{L}\left\{\int_0^t f(x)\,dx\right\} = \frac{1}{s}F(s)$$

$$\mathcal{L}\{f(t)*g(t)\} \equiv \mathcal{L}\left\{\int_0^t f(t-x)g(x)\,dx\right\} = F(s)G(s), \text{ where } G(s) = \mathcal{L}\{g(t)\}$$

$$\mathcal{L}\{\delta(t-a)\} = e^{-as}, \quad a \geq 0$$

$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt \text{ if } f \text{ is periodic with period } T$$

Table of Fourier Series

1. The Fourier series of a $2L$ periodic function f is given by

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right],$$

with

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos\left(\frac{n\pi x}{L}\right)\, dx = \frac{1}{L} \int_{\alpha}^{\alpha+2L} f(x) \cos\left(\frac{n\pi x}{L}\right)\, dx, \quad n \geq 0,$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right)\, dx = \frac{1}{L} \int_{\alpha}^{\alpha+2L} f(x) \sin\left(\frac{n\pi x}{L}\right)\, dx, \quad n \geq 1,$$

where α is any real number. If f is an odd function, then

$$a_n = 0 \quad \textit{and} \quad b_n = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi x}{L}\right)\, dx \quad n \geq 1.$$

If f is an even function, then

$$b_n = 0 \quad \textit{and} \quad a_n = \frac{2}{L} \int_0^L f(x) \cos\left(\frac{n\pi x}{L}\right)\, dx \quad n \geq 0.$$

2. The Fourier series of a function $f(x)$ defined on $[a, b]$ with $b - a = 2L$ is given by

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right],$$

with

$$a_n = \frac{1}{L} \int_a^b f(x) \cos\left(\frac{n\pi x}{L}\right)\, dx, \quad n \geq 0,$$

$$b_n = \frac{1}{L} \int_a^b f(x) \sin\left(\frac{n\pi x}{L}\right)\, dx, \quad n \geq 1.$$

If the $2L$-periodic extension $\tilde{f}$ of f to $\mathbb{R}$ is an odd function, then $a_n = 0$, and if $\tilde{f}$ is an even function, then $b_n = 0$.

3. The Fourier sine series of a function f defined on $[0, L]$ is given by

$$\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right), \quad b_n = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi x}{L}\right)\, dx, \quad n \geq 1.$$

4. The Fourier cosine series of a function f defined on $[0, L]$ is given by

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right), \quad a_n = \frac{2}{L} \int_0^L f(x) \cos\left(\frac{n\pi x}{L}\right)\, dx, \quad n \geq 0.$$

Table of Fourier Transforms

$\mathcal{F}\{f(x)\} = \hat{f}(\lambda) = \int_{-\infty}^{\infty} f(x)e^{i\lambda x}dx.$

$\mathcal{F}^{-1}\left\{\hat{f}(\lambda)\right\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{f}(\lambda)e^{-i\lambda x}d\lambda.$

$\mathcal{F}(u(x-a) - u(x-b)) = \frac{e^{i\lambda b}-e^{i\lambda a}}{i\lambda}, a < b.$

$\mathcal{F}(u(x+b) - u(x-b)) = \frac{e^{i\lambda b}-e^{-i\lambda b}}{i\lambda} = \frac{2\sin(\lambda b)}{\lambda}.$

$\mathcal{F}(e^{-|x|}) = \frac{2}{1+\lambda^2}.$

$\mathcal{F}\{e^{iax}f(x)\} = \hat{f}(\lambda + a).$

$\mathcal{F}\{f(x-a)\} = e^{ia\lambda}\hat{f}(\lambda).$

$\mathcal{F}(f'(x)) = -i\lambda\hat{f}(\lambda).$

$\mathcal{F}\{xf(x)\} = -i\frac{d\hat{f}(\lambda)}{d\lambda}.$

$\mathcal{F}(e^{-tx^2}) = \frac{\sqrt{\pi}}{\sqrt{t}}e^{-\lambda^2/(4t)}, t > 0.$

$\mathcal{F}\{f(\alpha x)\} = \frac{1}{|\alpha|}\hat{f}\left(\frac{\lambda}{\alpha}\right), \alpha \neq 0.$

$\mathcal{F}\{(f * g)(x)\} = \mathcal{F}\left\{\int_{-\infty}^{\infty} f(s)g(x-s)ds\right\} = \mathcal{F}(f(x))\mathcal{F}(g(x)).$
$\mathcal{F}\{\delta(x-a)\} = e^{i\lambda a}.$